Impact of Reference Point on Lose-Averse Newsvendor Problem

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Abstract: Newsvendor problem is a fundamental single-item inventory problem with stochastic demands over a single period. In the traditional newsvendor model, the expected utility theory is introduced to measure the risk attitude of decision-maker. However, behavior of decision-maker for many practical problems cannot be explained with the expected utility theory. As a result, prospect theory of the behavioral economics was adopted to describe the risk preference of decision-maker. In this paper, value function of the prospect theory was used to quantify the risk bias of the decision-maker. A widely accepted piecewise-linear form of value function was considered. The impact of the reference point on the optimal order quantity was, then, analyzed. The results show that the optimal order quantity of newsvendor problem on prospect theory is not a unique value. The order is in an interval varying with different reference point. Furthermore, several typical stochastic demand distribution density functions, for example, uniform distribution, exponential distribution were tested for the model. The results of numerical examples were consistent with some anomalistic findings of experiments in the literature.

Keywords: Newsvendor Problem; Lose-Aversion; Prospect Theory

I. Introduction

The newsvendor problem is a classical problem of controlling the level of the inventory of a single item with stochastic demands over a single period [1]. In this problem, a newsvendor tries to decide how many newspapers to stock on a newsstand before selling season. He only knew the stochastic distribution of demand in the period. Moreover, he has only one opportunity to place the order, and further replenishments are impossible. As a result, if the newsvendor orders too little, he loses sales; and if he orders too much, he must dispose of the excess stock at a loss. It is therefore the problem of deciding the size of a single order to balance the costs of overage against underage. The problem is particularly important for items with significant demand uncertainty and large both overage and underage costs.

The traditional newsvendor model is built with a risk-neutral decision-maker so that its objective is to maximize expected profit. Later, risk preference (risk seeking, risk neutral, and risk averse, etc) of decision-maker is included into the model. In this extension of the newsvendor model, the utility function from expected utility theory is introduced to describe different risk preference. For example, risk neutral decision-maker has a linear utility function, while risk seeking decision-maker has convex and risk averse has concave utility function. However, the expected utility theory fails to provide an acceptable explanation about many empirical results. It indicates that decision-makers do not always behave according to the assumptions of the expected utility theory. Therefore, prospect theory from behavioral economics is proposed to understand how people make choices from alternatives that involve risk.

In this paper, a loss-averse newsvendor model is presented, where value function is used to describe risk preference of decision-maker instead of utility function. Different from other related works, this paper focuses on impact of the reference point of the value function on the optimal solution.

The remainder of this paper is organized as follows. Section 2 reviews existing approaches related to our research. Section 3 describes our single-period loss-averse newsvendor mode. In section 4, we analyze the effects of changing the reference point of the value function. Numerical experiments are given in the Section 5. Finally, Section 6 concludes the study.

II. Related Works

There are increasing numbers of research papers examining the behavior of actual human agents in complex decision problems. In the inventory context, newsvendor problem is one of most studied [2]. In traditional approaches, utility function is adopted to reflect the subjective value of decision-maker for losses and gains. So, the objective of newsvendor problem is to maximize the expected utility value. For example, Keren et al. derived the first order conditions for optimality for the risk-averse newsvendor problem [3]; Agrawal et al. considered an extended risk-averse newsvendor problem, where the distribution of demand is a function of the selling price [4]; Eechkoudt et al. examined the effects of risk in risk-averse newsvendor problem [5]. In that paper, initial wealth of newsvendor functions in the same way as the reference point in the value function.

However, the expected utility theory failed to explain a lot of practical phenomena. For example, Schweitzer and Cachon described two experiments that investigate newsvendor decisions [6]. Results demonstrated that choices systematically deviated from those that maximize expected profit. Bolton's experiments also implied that the institutional organization of experience and feedback had a significant influence on whether inventory is stocked optimally [7]. These show that real-world decision-maker do not always act as a rational agent, and chooses the alternative according to rule of maximizing expected utility value. Therefore, some experts turned to prospect theory of behavioral economics to re-figure risk preference of decision-makers [8]. Wang et al. found that if shortage cost was not negligible, a loss-averse newsvendor may order more than a risk-neutral newsvendor [9]. Su investigated the effect of bounded rationality in newsvendor model [10]. In the model, decision-maker does not always choose the utility-maximizing alternative. Instead, alternative with higher utility is chosen with larger probability.

In this paper, value function from prospect theory is also proposed to study classical newsvendor problem. Different from existing approaches, we address the influence of the reference point on the optimal order quantity.

III. Lose-Averse Newsvendor Model

Prospect theory was developed by Kahneman and Tversky as an important alternative to the expected utility theory [8]. An essential assumption of the theory is that the carriers of value are changes in wealth or welfare, rather than final states. Thus, the value function has two arguments: 1) the asset position that serves as reference point; and, 2) the magnitude of the change (positive or negative) from that reference point. Moreover, given the same variation in absolute value, there is a bigger impact of losses than of gains. It implied that people usually prefer avoiding losses to acquiring gains, loss-averse named.

As mentioned above, increasingly researchers use the value function to measure losses and gains, instead of absolute wealth as the utility function does. However, this paper addresses to answer several more interesting questions:

1) Is optimal order quantity of the loss-averse newsvendor problem independent of reference point?
2) Is optimal order quantity of the lose-averse newsvendor problem unique?

If first answer is yes while the second is no, then,

3) Given an arbitrary reference point, what happens if optimal solution of that reference point is set as reference point of next iteration, and re-calculates the problem?

Firstly, the main notations used throughout the paper are stated below.

K	cost of each unit of products
P	selling price per unit, *P>K*
C	salvage value for unsold unit, *C<K*
r	demand
f(r)	distribution function of demand *r*
F(r)	cumulative distribution function of demand *r*
Q'	reference point
*Q**	optimal order quantity
V(x)	a value function
ρ	a constant, *=(K-C)/(P-C)*

Given *Q'*, if newsvendor orders more $q^+ (q^+>0)$ than *Q'*, the increased gain/loss is:

$$\pi^+(Q',q_+)=\begin{cases}-(K-C)\cdot q_+, & r<Q'\\ (P-C)\cdot(r-Q'-\rho\cdot q_+), & Q'\le r\le Q'+q_+\\ (P-K)\cdot q_+, & r>Q'+q_+\end{cases} \quad (1)$$

Similarly, if newsvendor orders less q-(q->0) than Q', the increased gain/loss is:

$$\pi^-(Q',q_-)=\begin{cases}(K-C)\cdot q_-, & r<Q'-q_-\\ (P-C)\cdot(Q'-(1-\rho)\cdot q_- - r), & Q'-q_-\le r\le Q'\\ -(P-K)\cdot q_-, & r>Q'\end{cases} \quad (2)$$

In this paper, we also consider a simple piecewise-linear form of loss-averse value function [9]:

$$V(W)=\begin{cases}W-W_0, & W\ge W_0\\ \lambda\cdot(W-W_0), & W<W_0\end{cases},(\lambda>1) \quad (3)$$

IV. The Effect of Reference Point

Since the demand is a random variable, the goal is to maximize the payoff function. If newsvendor orders more q^+, then

$$\begin{aligned}EV(Q',q_+)&=\int_0^{Q'}V[-(K-C)\cdot q_+]\cdot f(r)dr\\&+\int_{Q'}^{Q'+\rho\cdot q_+}V[(P-C)\cdot(r-Q'-\rho\cdot q_+)]\cdot f(r)dr\\&+\int_{Q'+\rho\cdot q_+}^{Q'+q_+}V[(P-C)\cdot(r-Q'-\rho\cdot q_+)]\cdot f(r)dr\\&+\int_{Q'+q_+}^{\infty}V[(P-K)\cdot q_+]\cdot f(r)dr\\&=-\lambda\cdot\int_0^{Q'}(K-C)\cdot q_+\cdot f(r)dr\\&-\lambda\cdot\int_{Q'}^{Q'+\rho\cdot q_+}(P-C)\cdot(Q'+\rho\cdot q_+-r)\cdot f(r)dr\\&+\int_{Q'+\rho\cdot q_+}^{Q'+q_+}(P-C)\cdot(r-Q'-\rho\cdot q_+)\cdot f(r)dr\\&+\int_{Q'+q_+}^{\infty}(P-K)\cdot q_+\cdot f(r)dr\end{aligned} \quad (4)$$

The first order condition of (4) for the optimal order quantity is:

$$F(Q'+q_+^*)=(1-\rho)-(\lambda-1)\cdot\rho\cdot F(Q'+\rho\cdot q_+^*) \quad (5)$$

Let q_L be a number that satisfies $F(q_L)=(1-\rho)/(1-\rho+\lambda\cdot\rho)$.

Proposition 1. For $0\le Q'<q_L$, the optimal order quantity increment $q_+^*>0$; otherwise, $q_+^*=0$.

Proof. Let $q_+^*=0$, then (5) can be written as $F(Q')=(1-\rho)-(\lambda-1)\cdot\rho\cdot F(Q')$. As a result, $F(Q')=(1-\rho)/(1-\rho+\lambda\cdot\rho)\Rightarrow Q'=q_L$.

Proposition 2. As reference point $Q'(0\le Q'\le q_L)$ increases, the optimal order quantity $Q^*(=Q'+q_+^*)$ decreases.

Proof. Let q_1^*, q_2^* are optimal order quantity increment according to two different reference points-$Q_1', Q_2'(Q_2' > Q_1')$, respectively. By (5),

$$\begin{aligned} &F\left(Q_1'+q_1^*\right)+(\lambda-1)\cdot\rho\cdot F\left(Q_1'+\rho\cdot q_1^*\right) \\ &=F\left(Q_2'+q_2^*\right)+(\lambda-1)\cdot\rho\cdot F\left(Q_2'+\rho\cdot q_2^*\right) \end{aligned} \tag{6}$$

Let $q_2\left(=Q_1'+q_1^*-Q_2'\right)$, it means that $Q_2'+q_2=Q_1'+q_1^*$. Then,

$$\begin{aligned} &F\left(Q_2'+q_2\right)+(\lambda-1)\cdot\rho\cdot F\left(Q_2'+\rho\cdot q_2\right) \\ &=F\left(Q_1'+q_1^*\right)+(\lambda-1)\cdot\rho\cdot F\left(Q_1'+\rho\cdot q_1^*+(1-\rho)\cdot(Q_2'-Q_1')\right) \\ &>F\left(Q_1'+q_1^*\right)+(\lambda-1)\cdot\rho\cdot F\left(Q_1'+\rho\cdot q_1^*\right) \\ &=F\left(Q_2'+q_2^*\right)+(\lambda-1)\cdot\rho\cdot F\left(Q_2'+\rho\cdot q_2^*\right) \end{aligned} \tag{7}$$

Thus, $Q_1'+q_1^*=Q_2'+q_2>Q_2'+q_2^*$.

If we set *Q'=0*, by (5)

$$F\left(q_+^*\right)=(1-\rho)-(\lambda-1)\cdot\rho\cdot F\left(\rho\cdot q_+^*\right) \tag{8}$$

Therefore, we get the optimal order quantity $Q^*=q_+^*$. Since the optimal order quantity q of traditional risk-neural newsvendor problem satisfies $F(q)=(1-\rho)$, it is larger than *Q**. This result is consistent with conclusions in [9].

Similarly, if newsvendor order less q-, then

$$\begin{aligned} EV\left(Q',q_-\right)&=\int_0^{Q'-q_-}V\left[(K-C)\cdot q_-\right]\cdot f(r)\,dr \\ &+\int_{Q'-q_-}^{Q'-(1-\rho)\cdot q_-}V\left[(P-C)\cdot\left(Q'-(1-\rho)\cdot q_--r\right)\right]\cdot f(r)\,dr \\ &+\int_{Q'-(1-\rho)\cdot q_-}^{Q'}V\left[(P-C)\cdot\left(Q'-(1-\rho)\cdot q_--r\right)\right]\cdot f(r)\,dr \\ &+\int_{Q'}^{\infty}V\left[-(P-K)\cdot q_-\right]\cdot f(r)\,dr \\ &=\int_0^{Q'-q_-}(K-C)\cdot q_-\cdot f(r)\,dr \\ &+\int_{Q'-q_-}^{Q'-(1-\rho)\cdot q_-}(P-C)\cdot\left(Q'-(1-\rho)\cdot q_--r\right)\cdot f(r)\,dr \\ &-\lambda\cdot\int_{Q'-(1-\rho)\cdot q_-}^{Q'}(P-C)\cdot\left(r-Q'+(1-\rho)\cdot q_-\right)\cdot f(r)\,dr \\ &-\lambda\cdot\int_{Q'}^{\infty}(P-K)\cdot q_-\cdot f(r)\,dr \end{aligned} \tag{9}$$

The first order condition of (9) for the optimal solution is:

$$F\left(Q'-q_-^*\right)=(1-\rho)\cdot\left[\lambda-(\lambda-1)\cdot F\left(Q'-(1-\rho)\cdot q_-^*\right)\right] \tag{10}$$

Let $F\left(q_U\right)=(\lambda-\lambda\cdot\rho)/(\lambda-\lambda\cdot\rho+\rho)$.

Proposition 3. For $Q'\geq q_U$, the optimal order quantity decrement $q_-^*>0$; otherwise, $q_-^*=0$.

Proposition 4. For $q_L\leq Q'\leq q_U$, the optimal order quantity $Q^*=Q'\left(q_-^*=0\right)$.

Proposition 5. As reference point $Q'\left(Q'\geq q_L\right)$ increases, the optimal order quantity $Q^*\left(=Q'-q_-^*\right)$ decreases.

Proposition 6. For $0\leq Q'\leq q_L$, the optimal order quantity *Q** is in the range *[q_L,q_U]*.

Proof.

$$\begin{aligned} &F\left(Q'+q_+^*\right)=(1-\rho)-(\lambda-1)\cdot\rho\cdot F\left(Q'+\rho\cdot q_+^*\right) \\ &\geq(1-\rho)-(\lambda-1)\cdot\rho\cdot F\left(Q'+q_+^*\right) \\ &\Rightarrow F\left(Q'+q_+^*\right)\geq\frac{(1-\rho)}{(1-\rho+\lambda\cdot\rho)}=F\left(q_L\right) \end{aligned} \tag{11}$$

And,

$$\begin{aligned} &F\left(Q'+q_+^*\right)-F\left(q_U\right) \\ &=(1-\rho)-(\lambda-1)\cdot\rho\cdot F\left(Q'+\rho\cdot q_+^*\right)-\frac{(\lambda-\lambda\cdot\rho)}{(\lambda-\lambda\cdot\rho+\rho)} \\ &=-\rho-(\lambda-1)\cdot\rho\cdot F\left(Q'+\rho\cdot q_+^*\right)+\frac{\rho}{(\lambda-\lambda\cdot\rho+\rho)} \\ &=-\frac{\rho\cdot(\lambda-1)\cdot(1-\rho)}{(\lambda-\lambda\cdot\rho+\rho)}-(\lambda-1)\cdot\rho\cdot F\left(Q'+\rho\cdot q_+^*\right)<0 \\ &\Rightarrow F\left(Q'+q_+^*\right)<F\left(q_U\right)\Rightarrow\left(Q'+q_+^*\right)<q_U \end{aligned} \tag{12}$$

According to Proposition 4, this optimal order quantity $Q^*\left(=Q'+q_+^*\right)$ is the final optimal order quantity.

Proposition 7. For $Q'\geq q_U$, the optimal order quantity *Q** is also in the range *[q_L,q_U]*.

Proof.

$$\begin{aligned} F\left(Q'-q_-^*\right)&=\lambda\cdot(1-\rho)-(\lambda-1)\cdot(1-\rho)\cdot F\left(Q'-(1-\rho)\cdot q_-^*\right) \\ &\leq\lambda\cdot(1-\rho)-(\lambda-1)\cdot(1-\rho)\cdot F\left(Q'-q_-^*\right) \\ &\Rightarrow F\left(Q'-q_-^*\right)\leq\frac{(\lambda-\lambda\cdot\rho)}{(\lambda-\lambda\cdot\rho+\rho)}=F\left(q_U\right) \end{aligned} \tag{13}$$

And,

$$\begin{aligned} &F\left(Q'-q_-^*\right)-F\left(q_L\right) \\ &=(1-\rho)\cdot\left[\lambda-(\lambda-1)\cdot F\left(Q'-(1-\rho)\cdot q_-^*\right)-\frac{1}{(1-\rho+\lambda\cdot\rho)}\right] \\ &=(1-\rho)\cdot\left\{(\lambda-1)\cdot\left[1-F\left(Q'-(1-\rho)\cdot q_-^*\right)\right]+1-\frac{1}{(1-\rho+\lambda\cdot\rho)}\right\} \\ &=(\lambda-1)\cdot(1-\rho)\cdot\left[1-F\left(Q'-(1-\rho)\cdot q_-^*\right)+\frac{\rho}{(1-\rho+\lambda\cdot\rho)}\right]\geq 0 \\ &\Rightarrow F\left(Q'-q_-^*\right)\geq F\left(q_L\right)\Rightarrow\left(Q'-q_-^*\right)=q^*\geq\left(q_L\right) \end{aligned} \tag{14}$$

Proposition 8. For $\lambda=1$, since $F\left(q_U\right)=F\left(q_L\right)=1-\rho$, the optimal order quantity *Q** is the same optimal solution of traditional risk-neural newsvendor problem.

So, this analysis answers our questions above: the optimal order quantity for loss-averse newsvendor problem is not unique and related with the initial reference point.

V. Numerical Examples

Uniform demand distribution

If demand r is the uniform distribution in [0,b], $f(r)=1/b,\left(r\in[0,b]\right)$. For $0\le Q'\le(1-\rho)\cdot b/(1-\rho+\lambda\cdot\rho)$, the optimal order quantity Q^* is calculated by (5):

$$Q^*=\frac{(1-\rho)\cdot b}{1+(\lambda-1)\cdot\rho^2}-\frac{\rho\cdot(\lambda-1)\cdot(1-\rho)}{1+(\lambda-1)\cdot\rho^2}\cdot Q' \quad (15)$$

For $Q'>(\lambda-\lambda\cdot\rho)\cdot b/(\lambda-\lambda\cdot\rho+\rho)$, the optimal order quantity Q^* is calculated by (10)

$$Q^*=\frac{(1-\rho)\cdot b}{1+(\lambda-1)\cdot\rho^2}-\frac{\rho\cdot(\lambda-1)\cdot(1-\rho)\cdot Q'}{1+(\lambda-1)\cdot\rho^2} \quad (16)$$

Setting $\rho=0.8$, $\lambda=2.5$, $b=1000$, Figure 1 illustrates the Q^*.

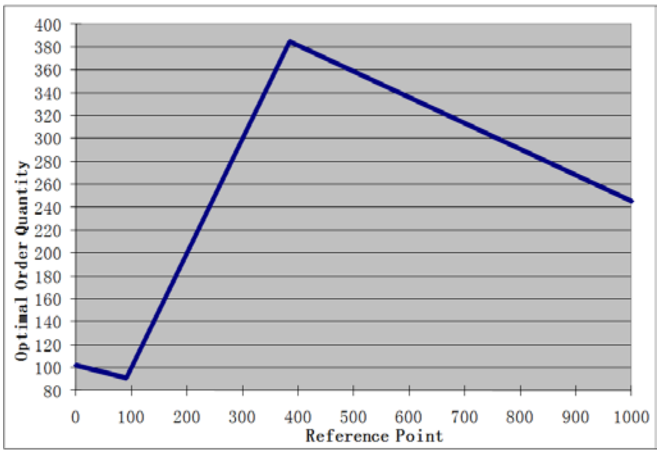

Figure 1. Q* under Uniform Demand Distribution

Figure 2 shows average optimal order quantity with different ρ, λ. From practical meanings of parameter ρ, product is more low-profit product when ρ is closer to 1. Figure 2 shows that the optimal order quantity of loss-averse newsvendor problem is larger than that of risk-neural newsvendor for low-profit products, while it is smaller for high-profit products [6].

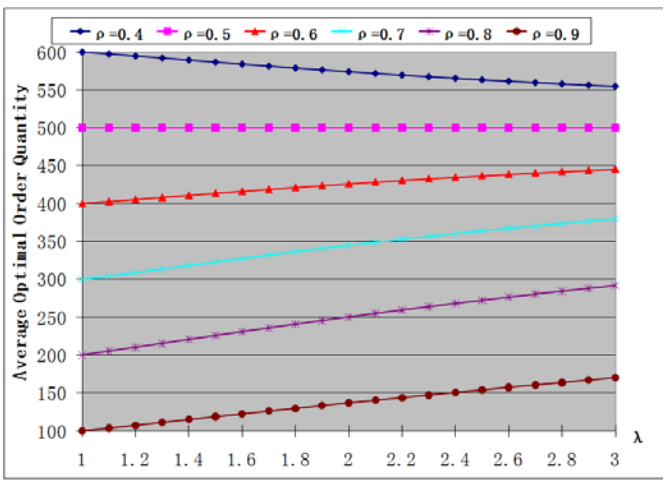

Figure 2. Average Q^* with Different ρ, λ

Exponential demand distribution

If demand r is the exponential distribution, $f(r)=\phi\cdot e^{-\phi\cdot r}$.

For $0\le Q'\le\left[\ln(1-\rho+\lambda\cdot\rho)-\ln(\lambda\cdot\rho)\right]/\phi$, $Q^*\left(=Q'+q_+^*\right)$ is calculated by (5)

$$\lambda\cdot\rho=e^{-\phi\cdot\left(Q'+q_+^*\right)}+(\lambda-1)\cdot\rho\cdot e^{-\phi\cdot\left(Q'+\rho\cdot q_+^*\right)} \quad (17)$$

For $Q'>\left[\ln(\lambda-\lambda\cdot\rho+\rho)-\ln(\rho)\right]/\phi$, $Q^*\left(=Q'-q_-^*\right)$ is calculated by (10)

$$\rho=e^{-\phi\cdot\left(Q'-q_-^*\right)}+(\lambda-1)\cdot(1-\rho)\cdot e^{-\phi\cdot\left(Q'-(1-\rho)\cdot q_-^*\right)} \quad (18)$$

Setting $\Phi=0.001$, Figure 3 shows Q^* under exponential demand distribution, and Figure 4 shows average optimal order quantity with different ρ, λ.

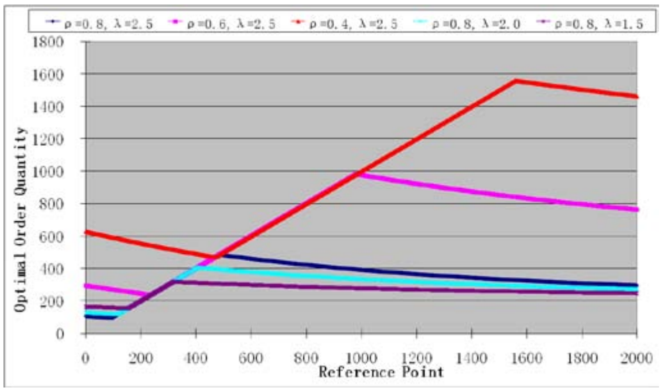

Figure 3. Q^* under Exponential Demand Distribution

In Figure 4, average Q^* is not a monotonic curve when ρ=0.45 or 0.5.

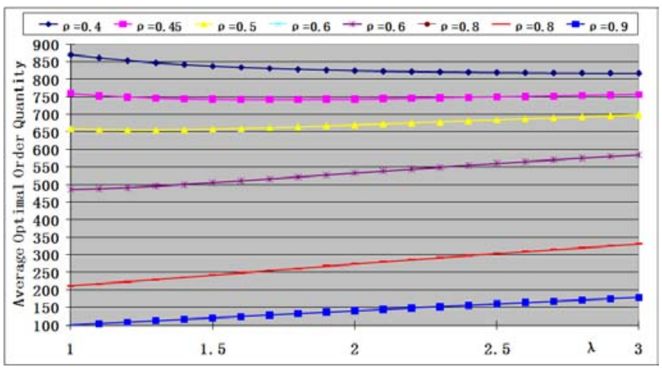

Figure 4. Average Q^* with Different ρ, λ

Comparing Q^* with two demands, Q^* is similar when reference point is small. However, Q^* with exponential demand is significantly greater than that with uniform demand when reference point increases.

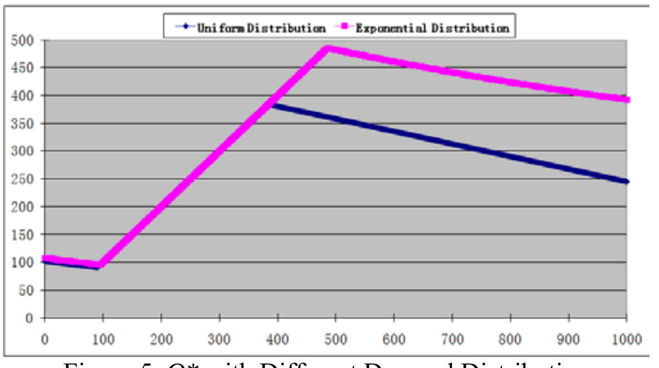

Figure 5. Q^* with Different Demand Distribution

VI. Conclusion

This paper address loss-averse newsvendor problem. A well-accepted piecewise-linear loss-averse value function is used to re-calculate optimal order quantity of newsvendor problem. We find that optimal order quantity is not unique, and related with reference point.

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